Errata to:

A Fixed-Point Farrago

February 15, 2018

Page 62 *Proof of* CLAIM: The second paragraph (proof of the "converse") does not work since the dot product of $x \in \Pi_M$ with itself need not be 1. The "proof of the converse" should go like this:

Thanks to Felix Werdermann and Felix Meitzner of the Free University of Berlin for pointing out this error.

"It remains to prove the converse. Write $\Delta := \sum_k \delta_k(x,y)$, and suppose $T_I(x,y) = x$, so that $\Delta(x,y)x = \delta(x,y)$, i.e. that,

$$\Delta(x,y)x_j = \delta_j(x,y) \qquad (j = 1,2,\dots M). \tag{*}$$

We wish to show that x is a best response to y, i.e., that $\delta(x,y)$ is the zero-vector. Suppose this is not the case, so $\delta_i(x,y) > 0$ for some index $i \in \{1,2,\dots M\}$. Thus $u_I(x,y) < u_I(e_i,y)$, i.e., $xAy^t < e_iAy^t$. Now for all the other indices j we have $u_j(x,y) \geq 0$, i.e., $xAy^t \leq e_jAy^t$, so upon multiplying both sides of this last inequality by x_j, noting that (*) insures $x_i > 0$, and using the fact that $\sum_j x_j = 1$, we see that

$$\left(\sum_{j=1}^{N} x_j e_k\right) Ay^t \; < \; xAy^t$$

But the sum in round brackets on the left is just the vector x, hence our assumption that $\delta(x,y) \neq 0$ has led to the contradiction $xAy^t < xAy^t$."

Page 139 **Lemma 11.16** (The Absorption Lemma): Incorrect statement and proof. The last paragraph of the proof does not show that $k = h^{n-m}$, merely that $h^n = k^m$.

Thanks to Ilja Gogic of University of Zagreb, Croatia for pointing out this error.

Thus the statement of Lemma 11.16 should include the additional assumption on G:

> *If $g \in G$ and $m \in \mathbb{N}$ then there are at most countably many $h \in G$ for which $g = h^m$.*

The circle group has this property since $g = h^m$ iff h is one of the m-th roots of g, so only m such h's qualify. □

The Absorption Lemma is used in the proofs of Thm 11.17 on page 139 and Thm. 11.19 on page 140. In both cases there is a line L through the origin such that the group G is the subgroup of rotations of $\mathbb{R}^3$ about the origin having L as axis of rotation; thus G is (isomorphic to) the circle group.

Page 139 A perhaps better statement and proof of the Absorption Lemma: For a group G, let $tG = \{g \in G\colon g^n = e \text{ for some } n \in \mathbb{N}\}$, the collection of elements of G having finite order. For G commutative, tG is easily seen to be a subgroup of G; it's called the "torsion subgroup." *Example:* The torsion subgroup of the circle group is the collection of elements $e^{2\pi i q}$ where $q \in \mathbb{R}$ is rational.

Revised Absorption Lemma. *Suppose X is a set, E is a subset of X, and C is a countable subset of E. Let G be a commutative group of self-maps of X that is fixed-point free on C, and for which each element takes C into E. If, in addition, G is uncountable, but its torsion subgroup is not, then E and $E \backslash C$ are G-equidecomposable.*

It's not required that each $g \in G$ take E into itself.

Revised proof. Only the last paragraph of the original proof needs to be modified. We wish to show that the set

$$H = \{g \in G\colon g^n(C) \cap C \neq \emptyset \text{ for some } n \in \mathbb{N}\}$$

is at most countable. To this end, note that $g \in H$ iff there exists a triple $(n, c, c') \in \mathbb{N} \times C \times C$ such that $g^n(c) = c'$. In other words, H is the union of the countable collection of sets

$$H_{n,c,c'} = \{g \in G\colon g^n(c) = c'\} \quad (n \in \mathbb{N}, c \text{ and } c' \in C).$$

Thus it's enough to show that each of the above sets is at most countable. For this, fix $g \in H_{n,c,c'}$. If $h \in H_{n,c,c'}$ we have $g^n(c) = c' = h^n(c)$, so

$$c = g^{-n}h^n(c) = (g^{-1}h)^n(c).$$

The second equality requires the commutativity of G.

We conclude from G's fixed-point free action on C that $(g^{-1}h)^n$ is the identity element of G, so $g^{-1}h \in tG$, and therefore $h \in g(tG)$. Thus $H_{n,c,c'}$ is a subset of $g(tG)$, so is at most countable, as we wished to show. □

Page 167 *Statement of Lemma 13.5*: "$K = \overline{\text{conv}}\, K$" should read "$K = \overline{\text{conv}}\, K_0$" .

Thanks to Christian Fenske for pointing out, in Zbl. Review # 1352.47002, both this error and the bibliography error below.

Page 214 *Bibliography entry* [99]: The volume should be 87 (not 67), and the year should be 1980 (not 2010).