Local Realism and Bell-Type Theorems

by Zafa Pi

The notions of locality and realism often arise in quantum mechanics (**QM**) literature. Both are assumptions made in classical physics. Locality states that no influence can be transmitted faster than light. Realism is a more subtle concept and is always tacit in classical theory. In QM texts the precise definition of realism and the consequences with respect to how calculations are performed usually involve the idea of hidden variables and non-elementary concepts in probability theory. The assumption of both locality and realism is called local realism (**LR**).

In 1935 Einstein, along with colleagues Podolsky and Rosen, gave an argument (assuming LR) that QM was deficient or incomplete, in that QM didn't take into account certain (hidden) variables that would allow predictions the theory was unable to do. Bohr contended their argument was deficient and QM was all that could be said about the phenomena in question; those predictions could not be made, the relevent data were random. All of the discussions were a bit fuzzy; neither side was able to convert the other, and no one was able to propose an experiment that would decide the issue.

In 1964 Bell concocted a brilliant (thought) experiment that could settle the matter, but the technology at that time was insufficient to pull it off. However, by the early '80s conclusive experiments were conducted; QM was vindicated and certain conclusions utilizing LR were shown to be inconsistent with experiment.

In what follows I will attempt to give an elementary definition of realism and show how LR is employed to give a short argument for Bell's inequality, along with two other refinements, all of which are inconsistent with QM.

We will have Alice and Bob each perform experiments that take no more than a minute to complete. They are to be separated by two light minutes and halfway between them there is a light station womanned by Eve. Eve simultaneously sends light signals to both. Alice and Bob each start their respective experiments when they receive Eve's signal. Locality says there is no transmission where one experiment can affect the other.

Let's call Alice's experiment A, and the result obtained from A to be a. Call Bob's experiment B, and the result he obtains b. So the joint result is [a,b]. Now suppose instead of B Bob did B* and got b*. The "reality" facing Alice as she is about to do A hasn't changed. What she observes is determined in some way by a real physical property of her experimental set up that exists just before her observation.

The existence of that "reality" is called realism. Thus the combination of both locality and realism, i.e. LR, allows us to conclude that we get the results [a,b] or [a,b*] depending on Bob's choice. *We get the same result, a, in both cases.*

N.B. some authors use the term locality in place of LR.

Here is a pertinent example: A is a very high coin flip on a windy day onto a bumpy surface; the result is heads (h). B is the same, with result tails. The joint result is [h,t]. If instead, B* was the roll of a die with result 5, the joint result would be [h,5]. How would one go about checking that h occurs in both cases? It can't be done, because there is no way to practically replicate A. Nevertheless, "in principle, if one knew all the variables (hidden or otherwise), i.e., the exact details of how the coin was flipped, the positions and velocities of the air molecules, etc. the value h could be refound". The stuff in quotes is classical talk. Physicists prior to QM wouldn't have even broached the idea of realism; it was tacitly assumed. Bohr didn't express doubts about locality, but he seemed to question realism.

(1) [Clauser, Horne] Now cometh the simplest Bell-type result I know of: Alice flips a fair coin to opt for experiment A or A*, and Bob independently flips to choose B or B*. The possible outcomes of all 4 experiments are r or y (for red or yellow).
The outcomes for [A,B] are [r,r], [r,y], [y,r], [y,y]; plus the following two provisos:

- If B yields value r then so does A*, e.g. [y,r] is not a possible result for [A*,B].
- If A yields r then so does B*.

How could the results of such experiments come about? What Eve could do is send colored light beams, a top beam and a bottom beam to each of Alice and Bob. The colors can be either red or yellow. To do experiment A or B they would merely note the color of their top beam; while doing experiment A* or B* they would observe the color of the bottom beam. Now Eve could, for instance, flip a fair coin to decide which color for each top beam. But, if she gets r for Alice's she makes Bob's bottom beam r as well, and vise versa. In all other cases she flips again for bottom beam colors. In this situation, after many pairs of experiments, they will see [r,r] about 25% of the time in the case [A,B] and see [r,r] about 56% when running [A*.B*] (check it out).

There are many ways (1) could be done, but using LR we always get the same conclusion.

Conclusion: If they had both run the experiment using [A,B] and obtained [r,r] they would have also would have obtained [r,r] using [A*,B*] instead. This is because if both A, and B had yielded r, but if Alice instead had flipped to use A*, then A* would have yielded r. This is because B would have still produced r (by LR), and the first proviso. Bob could do the same, so both A* and B* would have yielded r in this case. Hence if they try doing a zillion experiments *they should observe [r,r] at least as often for [A*,B*] as they do for [A,B].*

HOWEVER, employing QM techniques it's possible to satisfy the conditions of (1), with (r,r) occurring about 9% of the time for [A,B] and not occurring at all for [A*,B*] **!?!** What Eve sends to Alice and Bob is a particular pair of entangled photons (fauxtons may be a more appropriate term). What this means and what A, B, A*, and B* are and do can be made clear after a (my) short course in QM.

(2) Here comes the usual Bell Inequality: When they see the light Alice and Bob each flip a fair 3 sided coin (a die will work) with values x, y, and z. Alice then selects either experiment A_x, A_y, or A_z depending on the outcome of the flip. Same for Bob, with B's. The experiments A_x, ... , B_z all have +1, and -1 as possible outcomes.

Their goal is to design experiments such that the values for the outcomes, a_x, a_y, ..., b_z, of the experiments maximize their chances of getting different values for their experiment pair, with the proviso that $a_x = b_x$, $a_y = b_y$, $a_z = b_z$. (or $a_k = b_k$).

For example, If they agree to let $a_x = b_x = 1$, $a_y = b_y = 1$, and $a_z = b_z = -1$ then 4 out of the 9 equally likely possible pairs are different, namely,

$[a_x,b_z]$, $[a_y,b_z]$, $[a_z,b_x]$, $[a_z,b_y]$.

Big Question: Can they make their chances of success any better than 4/9?

Answer: No. The values for a pair are different if their product is -1. So the question comes down to how many of the $\{a_jb_k\} = \{a_xb_x, a_xb_y, ..., a_zb_z\}$ can be = to -1?
Since $a_k = b_k$, the sum of the a_jb_k = sum of $a_ja_k = (a_x + a_y + a_z)^2$.
Since a_k = +1 or -1, $a_x + a_y + a_z$ = +1 or -1 or +3 or -3, so $(a_x + a_y + a_z)^2 \geq 1$.
Since the sum of a_ja_k is ≥ 1, no more than 4 of the 9 a_ja_k can = -1. So their probability of success is $\leq 4/9$; this the Bell inequality.

(Notice we assumed LR since a_x is paired with both b_x and b_y)

BUT, Eve can send an entangled pair of photons off to Alice and Bob who employ ingeniously designed A's and B's that results in giving them a probability of 1/2 **!?!**

O God what have you wrought?

Yes my Lord, I concur with Albert that you are subtle, and perhaps not malicious; but you're also exasperating, impish, and a bit mischievous. Often you've been observed in some dark recess of the cosmos playing dice. Regularly you delete the existence of the moon when I'm not looking. And not only do you move in mysterious ways, you exhibit spooky action at a distance.

All of my conventional intuition you have placed in doubt, leaving me confounded. My only recourse is to deny your existence.

I now adopt atheism. Sayonara dude: **POOF**

And now the best of them all:

(3) [Greenberger, Horne, Zeilinger] In this ingenious experiment all probabilities are either 100% or 0%. Now we have Alice, Bob, Eve, and Fermi all mutually 2 light minutes apart, and Fermi turns on his light. As soon as Alice, Bob, and Eve see the light they each flip a fair coin. If Alice flips a head she chooses experiment A which can produce either +1 or -1. If she flips tails she chooses A* which also yields +1 or -1. The same goes for B, B*, E, E*.

We have the proviso that if exactly one of them flipped a head then either only one of Alice, Bob, or Eve produces +1, or all three of them do.

This can be accomplished in several ways; e.g., they all agree to produce +1 no matter what, or agree that if a person flips heads that person's experiment yields +1, and -1 if s/he flips tails.

Conclusion: If all three flipped heads their responses must be the same as if only one flipped a head.
To see why, let a be the number (+1 or -1) that A yields after Alice flips heads, and a* what A* yields after flipping tails. The same goes for b, b*, e, e*.
By the proviso, the product $ab^*c^* = 1 = a^*bc^* = a^*b^*c$ (notice that we used a* in two different experiments-LR). Thus
$(ab^*e^*)(a^*be^*)(a^*b^*e) = 1 = (abe)a^*a^*b^*b^*c^*c^*$. But $a^*a^* = b^*b^* = e^*e^* = 1$,
so $abe = 1$, which means either just one of a,b,e is 1 or they all are.

YET, if Fermi uses a slick triple of entangled photons and the devices A, A*, B, ... are artfully selected, then the proviso of (3) will be met, but with $abe = -1$ **!?!**

Every test of QM has panned out, and has done so with precision. The only tenable position one can take, given the above 3 experiments, is that local realism is incompatible with the workings of the "real world" (whatever delusion that may be). Therefore either locality isn't valid, or realism isn't valid, or neither is.

Some physicists are quite adamant as to which aspect of classical reality to deny and have constructed elaborate theories based on that choice. Others have chosen to deny the adequacy of our standard logic proposing theories of "quantum logic". Some of these efforts have allegedly led to new testable predictions, e.g., "Bohmian QM".
And then there is the vast lay literature of "quantum flapdoodle" justifying everything from ESP with living ETs to communicating with the dead. This stuff is not even wrong.

I find QM a wonderful testimonial to human endeavors; that such a beautiful, insightful, and practical theory could be constructed from the unintuitive and wayward shenanigans of invisibly small entities is truly amazing.